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STABILITY TESTS FOR ALPHAS AND BETAS OVER BULL AND BEAR MARKET CONDITIONS

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I. INTRODUCTION

THIS PAPER EMPLOYS standard econometric significance tests to determine whether the regression statistics from a sample of 700 NYSE stocks differ significantly when measured over bull and bear market conditions.[1] The need for such tests arises from several different sources. Levy suggested calculating separate beta coefficients for bull and bear markets [10]. Black has developed a two-factor market model which allows for the alpha to shift over time [1, 2]. Some investment advisors and large national brokerages have followed the advice of Levy and Black and sell separate alpha and beta statistics for bull and bear markets for aggregate fees (mostly in soft commission dollars) of millions of dollars per year.[2]

The specific subject of this inquiry is the single-index market model (SIMM) shown in equation (1).

$$r_{it} = a_i + b_i r_{mt} + e_{it} \tag{1}$$

where r_{it} denotes the capital gain plus cash dividend rate of return for the ith common stock in the tth month; r_{mt} represents the pre-tax capital gain plus cash dividend one-month rate of return calculated from Standard and Poor's 500 Composite (SP500) for the tth month, a_i and b_i are the regression intercept (called alpha) and slope (called beta) coefficients for the ith NYSE stock, and e_{it} is the unsystematic residual return.

Section II of this paper explains the econometric model formulated to test for bull and bear market changes in the statistics. Different definitions of bullish and bearish market conditions which are used in the tests are discussed in the third section. Section IV explains results of the test to see if the SIMM as a whole

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1. The tests for statistical significance used in this study are all based on the normal distribution. Fama [5] and others have shown evidence suggesting that one-month returns may be distributed according to a Paretian distribution with a characteristic exponent below two; and, as a result, tests based on the normal distribution may be biased. No adjustments were made for such possible bias, however, because (a) Fama has suggested using the normal distribution for empirical tests in Chapters One and Two of [6], and, (b) Wise has suggested that using returns from a Paretian distribution with a characteristic exponent below two creates minor problems at most [14].

2. A part-time faculty member at Baruch College who is a Vice President at a major brokerage informed one of the authors that the firm sold beta coefficients for total soft dollar remuneration of several million dollars in 1974 alone. The authors know of two large national brokerage firms which publish periodic (for example, monthly) books of beta statistics. Two other medium-sized national brokerages with which the authors are familiar also sell alpha and beta statistics in substantial volume.

changes in bullish and bearish market environments. Separate tests for the stability of the alpha intercept and the stability of the beta regression coefficient are presented in Section V. Section VI contains concluding remarks. An Appendix of supplementary statistics covering short-run and long-run samples is also provided at the end of the paper.

II. Test Formulation

Equation (2) is a modified version of the single-index market model (SIMM) shown in equation (1) which was formulated to test the stability of the alphas and betas over bull and bear market conditions.

$$r_{it} = A_{1i} + A_{2i} d_t + B_{1i} r_{mt} + B_{2i} d_t r_{mt} + u_{it} \qquad \text{Where } E(u_{it}) = 0 \tag{2}$$

The d_t variable in equation (2) is a binary variable which assumes the value of unity in bull markets and zero otherwise.[3] The coefficients on the binary variables, A_{2i} and B_{2i}, measure the differential effects of bull market conditions on the alpha, A_{1i}, and beta, B_{1i}, for the ith stock. But, over only bearish sample periods equation (2) reduces to equation (1) where $a_i = A_{1i}, b_i = B_{1i}$, and $e_{it} = u_{it}$ because $A_{2i} d_t = B_{2i} d_t = 0$. If the alpha and beta for the ith stock differ over bull and bear markets then A_{2i} and B_{2i} will be significantly different than zero.

III. Bull and Bear Market Definitions

The definition of bull and bear markets is of crucial importance in this inquiry. Since no sacrosanct definition exists the test was replicated using three alternative definitions of bull and bear market conditions, as explained below.

1. Bull and Bear (BB) Markets were delineated in accordance with the dates published in a popular investments textbook (see pages 464–5 of [4]). This procedure partitions the sampled months into two mutually exclusive and exhaustive subsets. It places most months when the market rises in the bullish category. But, months when the market rose admidst adjacent bearish months were classified as part of the bearish subset. That is, the BB categorization is based on market *trends*.[4]
2. Up and Down (UD) Markets. Months in which r_{mt} was non-negative are defined as Up months. And, months when r_{mt} was negative were placed in the Down category. This procedure yields a mutually exclusive and exhaustive division of the total sample into two subsets. But, the UD partitioning procedure ignores trends in the market and views every month independently.[5]
3. Substantial Up and Down (SUD) Months. The SUD procedure partitioned the sample into three subsets—(3a) months when the market moved Up-Substantially, (3b) months when the market moved Down-Substantially, and, (3c)

3. The test results are identical whether the zero-one dummy variable is set to unity for bull or for bear markets [7, p. 50].

4. There were 27 bear months from 1/66 through 9/66 and from 12/68 through 5/70. And there were 45 bull months from 10/66 through 11/68 and 6/70 through 12/71.

5. There were 40 Up months and 32 Down months.

months when the market moved neither Up- nor Down-Substantially.[6] Substantial moves were arbitrarily defined as months when the absolute value of r_{mt} was larger than half of one standard deviation of the market's returns measured over the total sampled period, that is, $|r_{mt}| > (.5\sigma_M)$. Months in which the market did not make Substantially-Up or Down movements were not used in the SUD estimates. That is, about one-third of the months were ignored because the market was essentially directionless under the SUD criteria which was employed.

Seventy-two months of data from January 1966 to December 1971 inclusive were taken from the CRSP Tape for the first 700 stocks which had continuous data over that period.[7] Equation (1) was estimated for each firm using all 72 monthly observations. At the 5% level of significance, 99.4% (that is, 696 firms) had significant betas (or $\bar{R}^2$'s). And, at the 1% level of significance 98.6% (that is, 690 firms) had significant beta (or $\bar{R}^2$) statistics. These 690 significant betas ranged from a low of .084 to a maximum of 2.370 with a mean value of 1.010.[8] The sample appears representative and suitable for further testing.

IV. Alpha and Beta Joint Test Results

If the distribution of returns which characterizes an equity share is altered by bull and/or bear market conditions, both the alpha and the beta may change simultaneously. One of the advantages of using equation (2) is that it yields simultaneous bull and bear market estimates of alpha and beta. Summary statistics for estimates of equation (2) based on the 700 security sample using three different definitions of bull and bear market conditions are presented below.

An *F*-test on the incremental sum-of-squares obtained by advancing from equation (1) to equation (2) was used to determine if the alpha and/or beta changed significantly with bull and bear market conditions [8, pp. 146 or 196–199]. The results of this test at both the 1% and 5% levels of statistical significance are reported in Table 1 below.

6. If $|r_{it}| < f\sigma_{mt}$ for $f > .5$ was used to filter out the more inconsequential months then the sample would have been too small. But, for $f < .5$ some months would have been included in the sample during which the market index made no substantial move. Thus, $f = .5$ was selected. For the value $f = .5$ there were 25 Up-Substantially months, 20 Down-Substantially months, and 27 sideways months in which the market did not move either Up- or Down-Substantially. The latter 27 observations were ignored in preparing the SUD estimates. This experiment was also replicated for the sample of returns with $|r_{it}| < (.5\sigma_{mt} + \bar{r}_m)$. However, the mean return on the market average was near zero and the results were essentially unchanged. In all cases, the SIMM, the alpha, and the beta experienced no more statistically significant changes than would be suggested by normal sampling theory.

7. This 72 month sample was selected primarily because it was the latest six-year sample which was categorized monthly into bull and bear market periods in an investments book over which the authors exercised no control (namely, pages 464–5 of [4]). However, the conclusions of beta stability reached in this paper were robust over samples as long as 15 years and as short as three years (which encompassed only one bull and one bear period). The conclusions were also unchanged (a) whether the SP500 Market Index or the Lorie-Fisher Index were used, and, (b) whether or not cash dividends were included in the rate of return calculations. See the Appendix for statistics for short-period and long-period tests.

8. Only 4 stocks had betas above 1.6 and only 4 stocks had betas below .4 out of the sample of 700 stocks for which equation (1) was estimated. Thus, ignoring these few outliers, the betas essentially ranged from .4 to 1.6 when measured over the 72 month sample.

TABLE 1

SUMMARY STATISTICS FOR EQUATION (2) ESTIMATED OVER 700 STOCKS

Bull and Bear Definition (obs.)	Significance Test	5%		1%	
		No.	Pc't.	No.	Pc't.
BB	A_{2i} and/or $B_{2i} \neq 0$*	43	6.2%	8	1.2%
(72 obs.)	$\bar{R}^2 \neq 0$**	691	98.7%	671	95.8%
UD	A_{2i} and/or $B_{2i} \neq 0$*	27	3.9%	7	1.0%
(72 obs.)	$\bar{R}^2 \neq 0$**	692	98.8%	672	96.0%
SUD	A_{2i} and/or $B_{2i} \neq 0$*	34	4.8%	7	1.0%
(45 obs.)	$R^2 \neq 0$**	693	99.0%	684	97.7%

* F-test on the incremental sum-of-squares obtained by advancing from equation (1) to equation (2). See pages 146 or 196–199 of [8].

** F-test to determine if at least one of the statistics—A_{2i}, B_{1i}, or B_{2i} are significantly different from zero. See page 143 of [8].

The test results in Table 1 are evidence which tends to indicate that the SIMM is not significantly different in bull markets than it is in bear markets. The few binary variables which were statistically significant were the approximate number that classic sampling theory suggests would be significant as a result of normal sampling errors.

V. SEPARATE TESTS OF ALPHAS AND BETAS

To determine if either the alpha or the beta coefficients were less stable between bull and bear market conditions the T-statistics of A_{2i} and B_{2i} in equation (2) for each of the 700 stocks were examined separately. If the ith stock's alpha intercept shifted significantly then the T-statistic for A_{2i} would be significantly different than zero. Or, if the beta regression coefficient changed then B_{2i} would be significant. Table 2 shows summary statistics of the tests for all 700 firms' alphas and betas.

The summary statistics in Table 2 for the separate T-tests on A_{2i} and B_{2i} from equation (2) tend to indicate that the betas may be less stable than the alphas as market conditions change from bullish to bearish. However, generally speaking,

TABLE 2

SUMMARY STATISTICS FOR SEPARATE T-TESTS ON ALPHAS & BETAS*

Bull and Bear Definition (obs.)	Coefficient Tested	5%		1%	
		No.	Pc't	No.	Pc't
BB	$T_{A2} \neq 0$	27	3.9%	5	.7%
(72 obs)	$T_{B2} \neq 0$	46	6.5%	11	1.6%
UD	$T_{A2} \neq 0$	34	4.8%	3	.4%
(72 obs)	$T_{B2} \neq 0$	35	5.0%	5	.7%
SUD	$T_{A2} \neq 0$	32	4.6%	3	.4%
(45 obs)	$T_{B2} \neq 0$	33	4.7%	7	1.0%

* Two-tailed T-test, see footnote one.

there were fewer T-statistics for both A_{2i} and B_{2i} which were significantly different from zero at both the 1% and the 5% levels of significance than sampling theory suggests would normally occur.[9] This is another shred of evidence that the SIMM is an econometric relationship which does not shift systematically with bullish and bearish market conditions.

Comparing the summary statistics in Tables 1 and 2 provides some additional suggestions that the SIMM is not affected by bull and bear markets. Note that the incidence of alphas or betas experiencing significant separate shifts (as evidenced in Table 2) tends to be less frequent than shifts in overall SIMM (as shown in Table 1). This fact indicates that some of the significant shifts in the SIMM merely result from marginally significant shifts in its statistics (most likely, the beta) rather than a genuine economic change.

Those few securities which had statistically significant coefficients on their binary variables were scrutinized to find a common thread which might relate them. For example, significantly different alphas and/or betas in bull and bear markets might be associated with stocks which had insignificant $\bar{R}^2$ for the SIMM,[10] were in a particular industry, ad infinitum. No commonality could be discerned however. These findings combined with the study by Klemkosky and Martin [9], which showed that most of the changes in the betas for individual stocks was attributable to random errors, underlines the futility of econometric efforts to differentiate between bull and bear market statistics for the SIMM.[11]

9. To test whether the percentage of significant dummy coefficients presented on Tables 1 and 2 are significantly different than the percentage expected from sampling theory, a t-test based on the binomial distribution was performed (see pages 547–556 of [15]). The null hypothesis is that the expected percentage is equal to the level of significance used to determine whether the estimated dummy coefficient(s) is significantly different than zero. The alternative hypothesis was that the observed percentage was greater than or less than the expected percentage. A one-tail test was employed. In only one case was the null hypothesis rejected. This occurred in the case where 6.5% of the B_2's were found to be significant with the 5% level of significance test. The probability of obtaining a difference of 1.5% between the observed and expected percentage is 2.8%. Hence, at the 5% level of significance, the null hypothesis is rejected and the observed discrepancy in percentages is statistically significant. This one small divergence between the actual results and the expected results suggests that the assumptions underlying the tests (namely, normality) are realistic.

10. The distinction between *any* change and a *statistically significant* change should be borne in mind. Stocks with a low R^2 for the SIMM sometimes have such large standard deviations around their alphas and betas that the beta could change from, say, .5 to 1.5 in bull and bear markets and still not have a statistically significant change. Stated more intuitively, this simply means that the alpha and beta estimates are no better than their goodness-of-fit statistics. The authors' unpublished research suggests the presence of random coefficients [12, Section 12.4] in equation (1) however.

11. This analysis also throws a ray of light onto the econometric interpretation of the returns from the so-called zero-beta-portfolio outlined in [1, 2]. The SIMM may be reformulated as shown in equation (3) since $a_i = r_{zt}(1 - b_i)$, where a_i is the alpha intercept from equation (1) and r_{zt} is the one-period return on the zero-beta-portfolio in the tth month.

$$r_{it} = r_{zt}(1 - b_i) + b_1 r_{mt} + q_{it} \qquad \text{where } E(q_{it}) = 0 \tag{3}$$

Since the alpha intercept in the SIMM does not shift significantly between bullish and bearish conditions this implies that r_{zt} does not shift with the degree of market optimism. If the returns on the zero-beta-portfolio are a significant econometrically observable phenomenon, some other economic rationale for the movements appears to be appropriate.

VI. Conclusions

The results above suggest that some investment analysts have fallen into the trap of misapplying econometric models and, as a result, are purveying erroneous information. Neither the alpha nor the beta statistics in the SIMM appear to be significantly affected by the alternating forces of bull and bear markets. Of course, the SIMM, the alpha, the beta and the statistics from all econometric models change from sample to sample. But, the question addressed here was whether or not these normal sampling errors were more than would occur in the classical econometric stability tests. Such instability would tend to depreciate the value of the received risk-return capital market theory [11]. However, the SIMM was found to be unaffected by the three different bull and bear market conditions which were delineated.

Appendix of Supplementary Statistics

The econometric tests described in the body of the paper were replicated over (a) one short sample period of 44 months from October 1966 through May 1970 inclusive, which included one bull and one bear market period (according to pages 464–5 of [4]), and, (b) one long sample period of 180 months from January 1950 through December 1964 inclusive, which included five bull and four bear markets (according to [4]). The results from these samples all support the conclusions in the body of the paper that the alpha, the beta, and the SIMM are stable over bull and bear markets. This supplementary evidence is shown below—it generalizes this study's stability conclusions to the short- and long-period investing horizons. Only statistics for the 5% level of significance and based on the Up-Down (UD) and Bull-Bear (BB) definitions (from [4]) are reported in this Appendix for the sake of brevity.

TABLE A1

Summary Statistics for Equation (2) Estimated For 700 NYSE Stocks, Long- and Short-Run Samples

Test Specification (Number of Obs.)	Statistics Tested	5% Level No.	5% Level Pc't.	Horizon
Bull-Bear Jan. 50 to Dec. 64 (180 months)	A_{2i} and/or $B_{2i} \neq 0$*	19	2.7%	Long
	$\bar{R}^2 \neq 0$**	700	100.0%	Long
Up-Down Jan. 50 to Dec. 64 (180 months)	A_{2i} and/or $B_{2i} \neq 0$*	46	6.6%	Long
	$\bar{R}^2 \neq 0$**	699	99.8%	Long
Bull-Bear Oct. 66 to May 70 (44 months)	A_{2i} and/or $B_{2i} \neq 0$*	41	5.8%	Short
	$\bar{R}^2 \neq 0$**	661	94.4%	Short
Up-Down Oct. 66 to May 70 (44 months)	A_{2i} and/or $B_{2i} \neq 0$*	29	4.1%	Short
	$\bar{R}^2 \neq 0$**	693	99.0%	Short

See footnotes beneath Table 1 about * and ** meanings.

TABLE A2

SUMMARY STATISTICS FOR SEPARATE T-TESTS ON ALPHAS & BETAS FOR 700 NYSE STOCKS, LONG- AND SHORT-RUN SAMPLES*

Bull and Bear Definition (obs)	Significance Test	5% Level No.	5% Level Pc't.	Horizon
Bull-Bear Jan. 50 to Dec. 64 (180 months)	$T_{A2} \neq 0$	25	3.6%	Long
	$T_{B2} \neq 0$	19	2.7%	
Up-Down Jan. 50 to Dec. 64 (180 months)	$T_{A2} \neq 0$	41	5.9%	
	$T_{B2} \neq 0$	34	4.9%	
Bull-Bear Oct. 66 to May 70 (44 months)	$T_{A2} \neq 0$	24	3.4%	Short
	$T_{B2} \neq 0$	57	8.1%	
Up-Down Oct. 66 to May 70 (44 months)	$T_{A2} \neq 0$	20	2.9%	
	$T_{B2} \neq 0$	25	3.6%	

* Two-tailed T-test, see footnote 1.

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